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# Dissipative analogies between a schematic macro-roughness arrangement and step–pool morphology

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## Abstract

We investigate flow resistance developed by macro-roughness represented by pebbles positioned on a granular layer according to a regularly spaced stripe pattern on steep bed slopes. Flume experiments under various geometrical and hydraulic conditions are carried out and interpreted by means of a theoretical model. Results show that flow resistance reaches a maximum and is due mainly to form drag when the spacing between macro-roughness stripes is about 10 times the average macro-roughness height. A statistical analysis based on various field observations of step–pool geometry underlines that this spacing appears to be one of the most frequent occurring in step–pool bed morphology sequences. Comparison between the present results and flow resistance evaluated for step–pools reproduced in the laboratory and observed in the field suggests that step–pool streams are characterized by a bed geometry able to develop the maximum flow resistance. Finally, a criterion is obtained to estimate flow resistance developed by natural step–pool streams when a formative flow discharge occurs based on geometric quantities only. Copyright © 2007 John Wiley & Sons, Ltd.

**Keywords:** step–pool; flow resistance; macro-roughness; flume experiments

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## Introduction

Step–pools are characteristic bedforms that are common in steep mountain streams with gradients between 3 and 30%, and incipient step–pool sequences have been observed for slopes down to 1% (Montgomery and Buffington, 1997; Lenzi, 2001). The step–pool pattern plays a fundamental role in mountain stream hydraulics, developing the major portion of flow resistance (Abrahams *et al.*, 1995). This flow resistance is essentially due to a loss of kinetic energy, dissipated by roller eddies that occur when water flows over a step rise and plunges into the pool below, creating a tumbling flow (Peterson and Mohanty, 1960). As a result, much of the energy that might otherwise be available for sediment erosion is lost, providing a bed stabilization (Ashida *et al.*, 1976). Following this idea some authors (e.g. Davies and Sutherland, 1980; Whittaker and Jaeggi, 1982; Abrahams *et al.*, 1995) have suggested that step–pool streams evolve toward a maximum flow resistance condition in order to achieve maximum bed stabilization. In particular Abrahams *et al.* (1995) suggest that maximum flow resistance occurs when  $1 < (H/L)/S < 2$ , where  $H$  is the step height,  $L$  the step–pool wavelength and  $S$  the average bed channel slope. More recent field investigations (Wohl *et al.*, 1997; Chartrand and Whiting, 2000; Zimmermann and Church, 2001) have somewhat extended the validity of the results of Abrahams *et al.* (1995), suggesting that where average channel slope is below 5–7% the  $(H/L)/S$  ratio is greater than two. Although several field and laboratory studies of step–pool geometry have been carried out (Whittaker and Jaeggi, 1982; D'Agostino and Lenzi, 1998; Chin, 1999; Chartrand and Whiting, 2000; Lenzi, 2001; Zimmermann and Church, 2001; Curran and Wilcock, 2005), very few data (Lee and Ferguson, 2002; Aberle and Smart, 2003; Curran and Wilcock, 2005) are available about the behaviour of hydraulic quantities and so very little is known about the flow resistance developed by step–pool streams. The lack of hydraulic data may be due to the difficulties of measuring low-frequency high-magnitude formative discharges, especially when step–pool sequences occur in rugged, mountainous and often inaccessible terrain (Chin, 1999).

In the present work we use theoretical analysis and laboratory experiments to investigate flow resistance when macro-roughness elements are positioned according to a regularly spaced stripe pattern on a layer of relatively fine material. Experiments are carried out in the case of no sediment transport changing the size of macro-roughness

elements, the spacing of the macro-roughness stripes, the flow discharge and the slope of the flume. Experimental results are interpreted by means of a theoretical model in order to investigate the behaviour of flow resistance as a function of hydraulic variables and geometric properties of macro-roughness stripes.

The aim of the work is to investigate the hypothesis suggested by Abrahams *et al.* (1995) of maximum flow resistance developed by natural step-pool sequences. We conducted our study to evaluate the behaviour of the flow resistance in its components of skin friction and form drag as a function of the main geometrical quantities (step-pool spacing  $H/L$  and slope  $S$ ). Although similar problems have been already tackled (for instance, Johnson and Le-Roux (1946) and Wohl and Ikeda (1998) carried out flume studies with artificial roughness elements, made of rectangular-section stripes on a flat bed), in the present experiments step-pool sequences are reproduced with a more realistic, although still very schematic, approximation. The dissipative analogies between the present experiments and natural step-pool sequences allow us to clarify the mechanisms associated to the development of the flow resistance.

A secondary objective is to obtain a predictive relationship for the flow resistance in natural step-pool sequences when the formative discharges occur. The existing formulae (see, e.g., Lee and Ferguson, 2002; Aberle and Smart, 2003) are typically based on some measure of the relative roughness and therefore need the evaluation of the mean flow depth, which can be difficult to measure. In the present work we develop a predictive relationship based on only geometrical quantities.

It is worth noting that the theoretical model and the experimental activity here recalled are the main aims of another paper (Canovaro *et al.*, 2007, accepted), where they are presented and discussed in detail.

## Theoretical Background

Flow resistance is herein evaluated by means of a theoretical model developed by Canovaro (2005) and Canovaro *et al.* (2007, accepted); a brief outline of the analysis is reported here.

The dimensionless Chezy coefficient  $C$  is calculated from the average longitudinal velocity in the case of steady flow and spatially averaged uniform conditions.  $C$  is defined as follows:

$$C = \frac{C'}{\sqrt{g}} = \sqrt{\frac{8}{f}} \quad (1)$$

with  $C'$  the classical dimensional Chezy coefficient,  $g$  gravity and  $f$  the Darcy–Weisbach friction factor.

As suggested by many authors (e.g. Marchand *et al.*, 1984; Bathurst, 1987), in the case of free-surface flow over macro-roughness elements (either vegetation or sediments) the assumption of a logarithmic velocity profile is no longer valid and the classical expression of flow resistance formulae cannot be applied. In order to obtain the mean longitudinal velocity, the flow region is assumed to be composed by two layers (Figure 1). In the bottom layer, which contains the major part of the macro-roughness elements lying on a granular layer, the flow is dominated by the loss of momentum due to the drag around the obstacles. In this layer of thickness  $d$  the flow velocity  $V_b$  is relatively low, and approximately constant over the depth. The upper layer extends up to the free surface and has a thickness  $Y - d$ , with  $Y$  the total flow depth always measured from the granular layer. In the upper layer, characterized by a mean flow velocity  $U$ , the flow velocity is relatively high and the major part of the flow discharge occurs. According to the two-layer approach herein adopted, the mean flow velocity profile is characterized by four variables:  $Y$ ,  $U$ ,  $d$  and  $V_b$ . These quantities allow us to estimate flow resistance in the case of macro-roughness flow conditions.

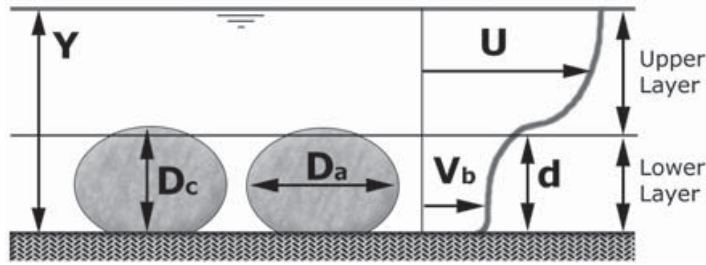
The macro-roughness elements are positioned with the long axis  $D_a$  in the direction parallel to the main flow, and the short axis  $D_c$  perpendicular to the bed; in the following calculation the pebbles are assumed to have an ellipsoidal shape. With regard to the orientation, Zimmermann and Church (2001) note that in nature  $D_a$  tends to be perpendicular to the flow; this feature would not change the present analysis since the obstacles are arranged along stripes across the flume and the drag component of the flow resistance is mainly related to the height of the stripes.

Considering a control volume of unit bed area extending from the bed to the water surface, the two governing equations, continuity and momentum balance, read as follows:

$$q = (Y - d)U + d\psi V_b \quad (2)$$

$$F_g = F_d + F_s \quad (3)$$

where  $q$  is the discharge per unit width,  $F_g$  the streamwise water mass weight,  $F_d$  the macro-roughness-induced drag force,  $F_s$  the surface-induced friction and  $\psi$  a reduction coefficient ( $\psi < 1$ ) for the bottom layer flow velocity  $V_b$ .  $\psi$  is



**Figure 1.** Sketch of a free surface flow over a bed with macro-roughness elements, showing notation used in the paper.

evaluated as the ratio between the effective volume occupied by the water among the macro-roughness elements in the lower layer and the total volume of the lower layer. Therefore,  $\psi V_b$  denotes the mean (apparent) flow velocity in the lower layer. It is worth noticing that the  $\psi$  coefficient depends on the effective bed arrangement. For a regularly spaced stripe arrangement of the macro-roughness, the reduction factor is estimated as follows (Canovaro *et al.*, 2007, accepted):

$$\psi = 1 - D_a D_b N_u \quad (4)$$

where  $D_b$  is the median axis diameter of macro-roughness elements and  $N_u$  is the number of macro-roughness elements arranged on a unit bed area along the channel.

The streamwise water mass weight component per unit bed area involved in Equation (3) is expressed as follows:

$$F_g = \rho g Y S \left( 1 - \frac{2 D_c \Gamma}{3 Y} \right) \quad (5)$$

where  $\rho$  is the water density,  $S$  is the channel slope and  $\Gamma$  is the spatial density (defined as the ratio between the number of macro-roughness elements arranged in a unit bed area and the maximum number that is possible to arrange in the same area:  $\Gamma = N_u / N_{\max}$ ). In Equation (5) the term in brackets acts as a reduction factor in order to exclude from calculations the control-volume fraction occupied by macro-roughness elements. In particular, the net volume of water  $V_w$  inside the control volume is evaluated as follows:

$$V_w = Y \times 1 \times 1 - \frac{\pi}{6} (D_a D_b D_c) N_u = Y \times 1 \times 1 - \frac{\pi}{6} (D_a D_b D_c) \Gamma \frac{1 \times 1}{\frac{\pi}{4} (D_a D_b)} \quad (6)$$

The drag force per unit bed area is

$$F_d = \frac{1}{2} \rho C_D N_u A_f' V_b^2 \quad (7)$$

where  $C_D$  is the drag coefficient and  $A_f'$  is the nearly ellipsoidal cross-section area of a single pebble perpendicular to the flow of interest to  $V_b$ , here approximated as follows:

$$A_f' = \frac{\pi}{4} D_b d. \quad (8)$$

Note that in Equation (8) the effective lower layer velocity  $V_b$  is employed instead of the apparent velocity  $\psi V_b$ , being a more appropriate characteristic velocity for evaluating the macro-roughness element drag. The drag coefficient  $C_D$  is herein assumed to be constant and equal to 1.5 (Robertson *et al.*, 1974; Bathurst, 1996).

Finally, the surface friction per unit bed area takes the following expression:

$$F_s = \rho U^2 \left[ \frac{(1 - \Gamma)}{C_G^2} + \frac{\Gamma}{C_p^2} \right] \quad (9)$$

where  $C_G$  and  $C_p$  are the Chezy coefficients of a channel bed without macro-roughness elements ( $\Gamma = 0$ ) and completely covered by macro-scale roughness ( $\Gamma = 1$ ), respectively;  $C_G$  and  $C_p$  were estimated from experiments.

In the present study  $Y$  and  $U$  are obtained by flume measurements, while  $d$  and  $V_b$  are unknowns to be determined by solving the equation system composed by Equations (2) and (3).

The mean flow velocity is here estimated as a weighted average of the flow velocities in the upper and bottom layers with weights equal to the flow discharges in the upper and bottom layers, and reads

$$\bar{U} = \frac{(Y - d)U^2 + d(\psi V_b)^2}{q} \quad (10)$$

The definition of mean velocity  $\bar{U}$  proposed here is consistent with the flow depth  $Y$ , which is always measured from the granular layer. When the bed is completely covered by the macro-roughness,  $V_b$  is very low and  $d$  is comparable with  $D_c$ ; in this condition the mean flow velocity  $\bar{U}$  is similar to  $U$ ; a classical depth-averaged value of the flow velocity would lead instead to an unrealistic mean value much smaller than  $U$ .

The dimensionless Chezy coefficient is then evaluated with the following:

$$C = \frac{\bar{U}}{\sqrt{F_g/\rho}} \quad (11)$$

## Laboratory Experiments

The experiments were carried out in a 10 m long, 36.5 cm wide and 50 cm deep glass-walled recirculating tilting flume. The measuring reach was 4 m long with a bed covered by a layer of granular uniform material having a median size of 7 mm.

In order to dissipate the disturbances and render the flow as uniform as possible, we placed a transition reach composed by small quarry rubble before the beginning of the measuring reach (Figure 2). Three different sets of pebbles with different sizes were used as macro-roughness elements (see Table I). Macro-roughness elements were positioned according to a regularly spaced stripe pattern along the experimental reach. Each stripe consisted of pebbles, placed over the granular layer with the long axis  $D_a$  in the flow direction and the short axis  $D_c$  perpendicular to the bottom of the flume.

For each set of pebbles the 'wavelength'  $L$ , i.e. the spacing between two consecutive stripes, was varied from 6 to 160 cm. Experiments were carried out with various discharges per unit width (13.0 to 38.4 l/s/m), and the bed slope was set between 1.0 and 6.0%. No sediment transport occurred during the experiments.

For each run both total water depth  $Y$  and flow velocity  $U$  were measured; several time-averaged values of these quantities were taken during each run along the measuring reach. Water surface elevation was measured by means of a set of 19 piezometers placed below the granular layer (see the sketch in Figure 3); by averaging these 19 values one space-time averaged water depth value is obtained for each run. Flow velocity was taken at 33 points according to a

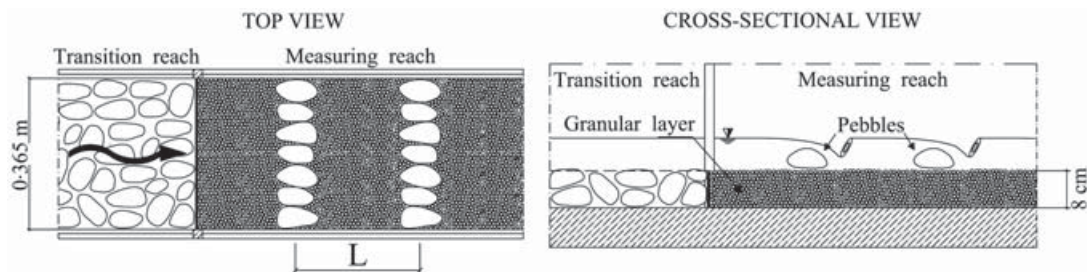
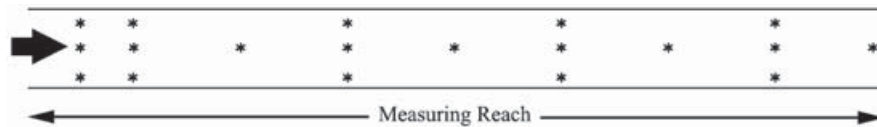


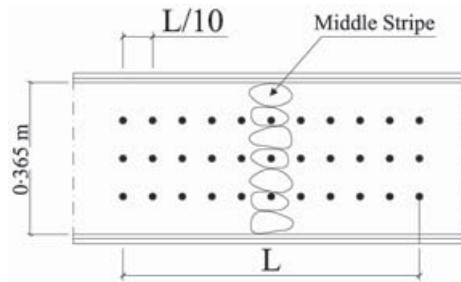
Figure 2. Sketch of the experimental set-up.

Table I. Mean geometrical properties of the pebbles employed

Set of pebbles	$\Phi$	$D_a$ [cm]	$D_b$ [cm]	$D_c$ [cm]
1	-5.25	5.5	3.8	2.6
2	-5.75	8.0	5.4	4.3
3	-6.25	9.8	7.6	5.3



**Figure 3.** Sketch of the piezometer inlets installed on the channel bottom (the arrow indicates the flow direction).



**Figure 4.** Sketch of the points of velocity measurement.

**Table II.** Characteristic parameters of the experiments

Experimental series	$\Phi$ [ - ]	$S$ [ - ]	$q$ [l/s/m]	$L$ [m]
I	-5.25	0.025	20.0	0.06-0.6
II	-5.75	0.010	20.0	0.08-0.8
III	-5.75	0.010	38.4	0.08-0.8
IV	-5.75	0.025	38.4	0.08-0.8
V	-6.25	0.025	38.4	0.10-0.8
VI	-5.75	0.025	13.0	0.08-1.6
VII	-5.75	0.025	20.0	0.08-1.6
VIII	-5.75	0.030	13.0	0.08-1.6
IX	-5.75	0.030	20.0	0.08-1.6
X	-5.75	0.030	22.6	0.08-1.6
XI	-5.75	0.040	14.3	0.08-1.6
XII	-5.75	0.040	17.4	0.08-1.6
XIII	-5.75	0.040	20.0	0.08-1.6
XIV	-5.75	0.040	22.6	0.08-1.6
XV	-5.75	0.060	17.4	0.08-1.6
XVI	-5.75	0.060	20.0	0.08-1.6
XVII	-5.75	0.060	22.6	0.08-1.6

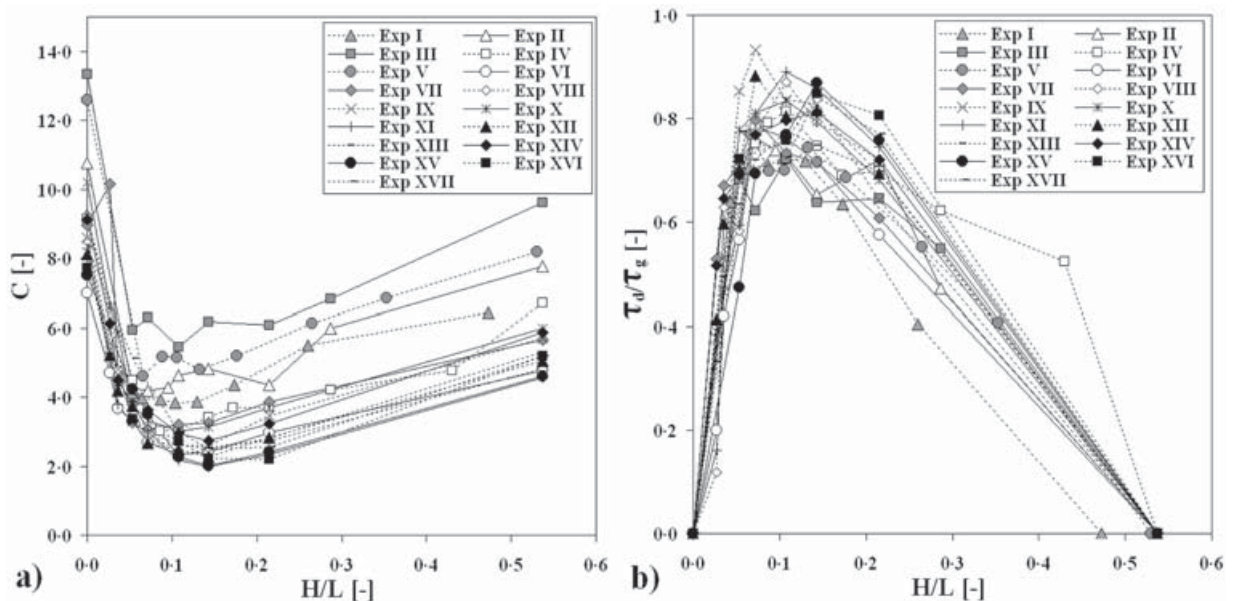
regular distribution along one wavelength across the middle stripe (Figure 4), by means of a micro-propeller meter. The flow velocity was measured in proximity to the water surface at about 80% of the depth from the bed by means of a micro-propeller. We made just one measurement along each vertical. As for the water elevation measurement, one space-time averaged velocity value is obtained for each run. Each value is the result of 400 velocity time-averaged readings, corresponding to a measurement time of 10 seconds.

The experimental programme involved a total of 160 runs divided into 17 experiments characterized by different stripe height, slope or discharge. A summary of the performed experiments is reported in Table II.

## Experimental Results

In Figure 5(a) and (b) the dimensionless Chezy coefficient ( $C$ ) and the ratio of drag shear stress ( $\tau_d$ ), defined as the drag force  $F_d$  per unit bed area, to total shear stress ( $\tau_g$ ), defined as the total force  $F_g$  per unit bed area, are presented



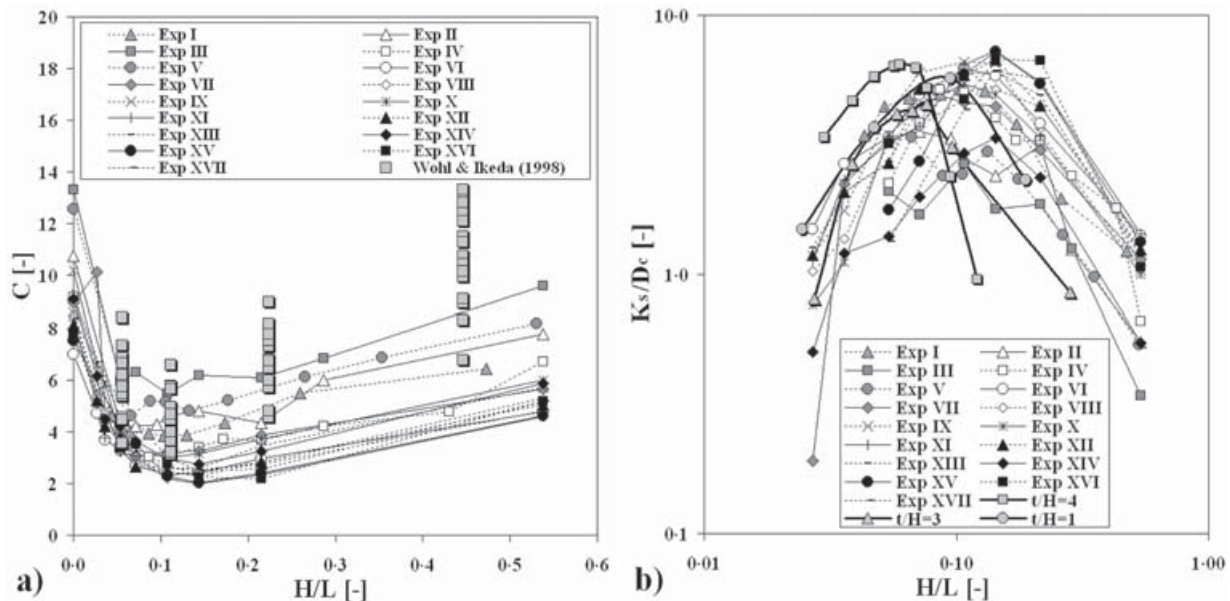


**Figure 5.** (a) Chezy coefficient ( $C$ ) and (b) ratio of drag shear stress ( $\tau_d$ ) to total shear stress ( $\tau_g$ ) as functions of ratio of stripe height ( $H$ ) to stripe wavelength ( $L$ ).

as functions of the ratio of stripe height ( $H$ ), taken equal to  $D_c$ , to macro-roughness stripe wavelength ( $L$ ). Both plots show a non-monotonic behaviour of the investigated quantities for all the experimental series. In particular, it is apparent that there is an 'optimal' dimensionless spacing value ( $H/L$ ), ranging around 0.1, which maximizes both flow resistance and the ratio of drag shear stress to total shear stress. When maximum flow resistance conditions are attained  $C$  is about two to three times smaller than in the absence of macro-roughness ( $H/L = 0$ ), but note that  $C$  is also smaller than when the spacing between stripes is minimum ( $L = D_a$ ). When flow resistance is maximum, drag shear stress is about 80% of total shear stress, suggesting that in this condition stripes play a fundamental role in energy dissipation while surface resistance plays an almost negligible role. Notwithstanding the presence of some scattering in the data points, it appears that flow discharge, flume slope and stripe height seem to have a second order effect on the value of the observed 'optimal' dimensionless spacing  $H/L$ . On the other hand, for a given  $H/L$ , flow resistance seems to increase with stripe height and bed slope while it decreases with flow discharge (compare Figure 5(a) with Table II).

The existence of an 'optimal'  $H/L$  value maximizing flow resistance is in agreement with the results of other investigations (e.g. Johnson and Le-Roux, 1946; Wohl and Ikeda, 1998) that studied the influence on flow resistance of macro-roughness positioned according to a regularly spaced stripe pattern, under various geometric and flow conditions. Figure 6 compares the present results with those of Wohl and Ikeda (1998) (a) and Johnson and Le-Roux (1946) (b) when macro-roughness composed by transverse rectangular ribs (in the latter experiments  $H$  is the height of the ribs and  $t$  the width of the ribs in the flow direction) is presented; note that in the latter case the equivalent roughness  $K_s$  parameter has been estimated as a measure of the flow resistance. The comparisons show a fairly good agreement; in particular, the presence of an 'optimal' spacing value maximizing flow resistance is revealed in both cases when  $H/L$  is around 0.1 as found in the present study. These findings suggest that the present experiments are representative of a common behaviour in the dissipative mechanism associated with regularly spaced stripes.

The presence of a maximum in flow resistance has also been found for macro-roughness spatial arrangements different from the one employed in the present study. Rouse (1965) indicates, in the case of a regular pattern of macro-roughness disposition, that the maximum flow resistance is achieved when macro-roughness spatial density (the sum of all basal areas of the macro-roughness elements arranged in a unitary bed area) is between 0.15 and 0.20. As pointed out by Morris and Wiggert (1971) for flow in closed conduits, the presence of a maximum in flow resistance may be explained in terms of a different behaviour of flow motion in relation to the spacing (or spatial density) of macro-roughness stripes. For high spacing values, when macro-roughness stripes are 'isolated', motion might be regarded as 'non-wake-interfering', while for low spacing values the vortex generation and dissipation associated with a stripe is not complete before the flow meets the next stripe.



**Figure 6.** Comparison with experiments of Wohl and Ikeda (1998) (a) and Johnson and Le-Roux (1946) (b). In the case of the latter comparison, the  $t/H$  ratio is taken around two, with  $t$  corresponding to  $D_o$  and  $H$  corresponding to  $D_c$ .

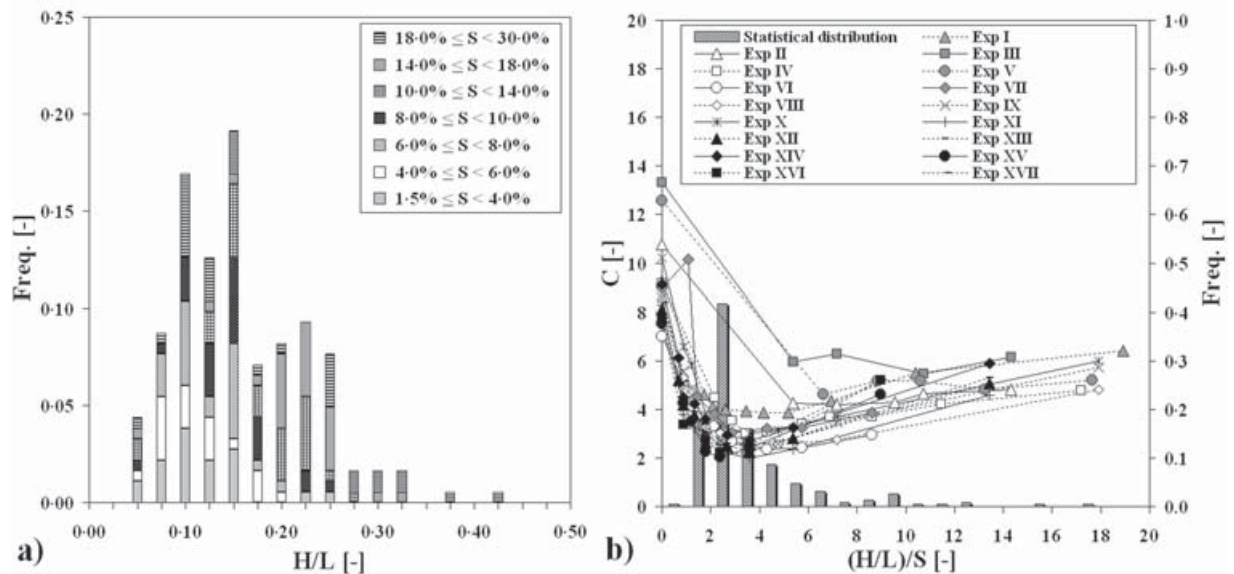
## Discussion

As shown in the previous section, maximum flow resistance in the case of regularly spaced stripes consists almost entirely of form resistance, associated with the plunging of flow downstream from each stripe and the formation of roller eddies in the pool between two successive stripes. This mechanism can be commonly observed in natural step-pool sequences (Abrahams *et al.*, 1995; Chin, 1998; Chartrand and Whiting, 2000), except for extremely high flow discharges, when the step-pool shape is hidden. Even if the present regular spaced stripe pattern is only a rough schematization of a natural morphology, in the light of the above findings it seems reasonable to consider some general analogies between flow resistance developed by natural step-pool sequences and the laboratory results presented herein.

Figure 7(a) shows the statistical frequency distribution of natural step-pool gradient ( $H/L$ ) based on 183 field and laboratory data on step-pool geometry, obtained from various authors (Whittaker and Jaeggi, 1982; Abrahams *et al.*, 1995; D'Agostino and Lenzi, 1998; Chin, 1999; Chartrand and Whiting, 2000; Lenzi, 2001; Zimmermann and Church, 2001; Curran and Wilcock, 2005). In the same plot the slope  $S$  is reported for each class of  $H/L$ . It appears that  $S$  ranges between 1.5% (data from Chartrand and Whiting, 2000) and 30% (data from D'Agostino and Lenzi, 1998); the larger slopes are generally associated with larger step-pool gradients ( $H/L > 0.25$ ) while smaller slopes are typical of smaller step-pool gradients ( $H/L < 0.1$ ). Moreover, from Figure 7(a) it can be seen that the range of most common spacing values observed in natural step-pool sequences is very close to the 'optimal' spacing value that maximizes flow resistance in the present flume experiments. This finding suggests using in the following part of this work only those flume runs that gave maximum flow resistance.

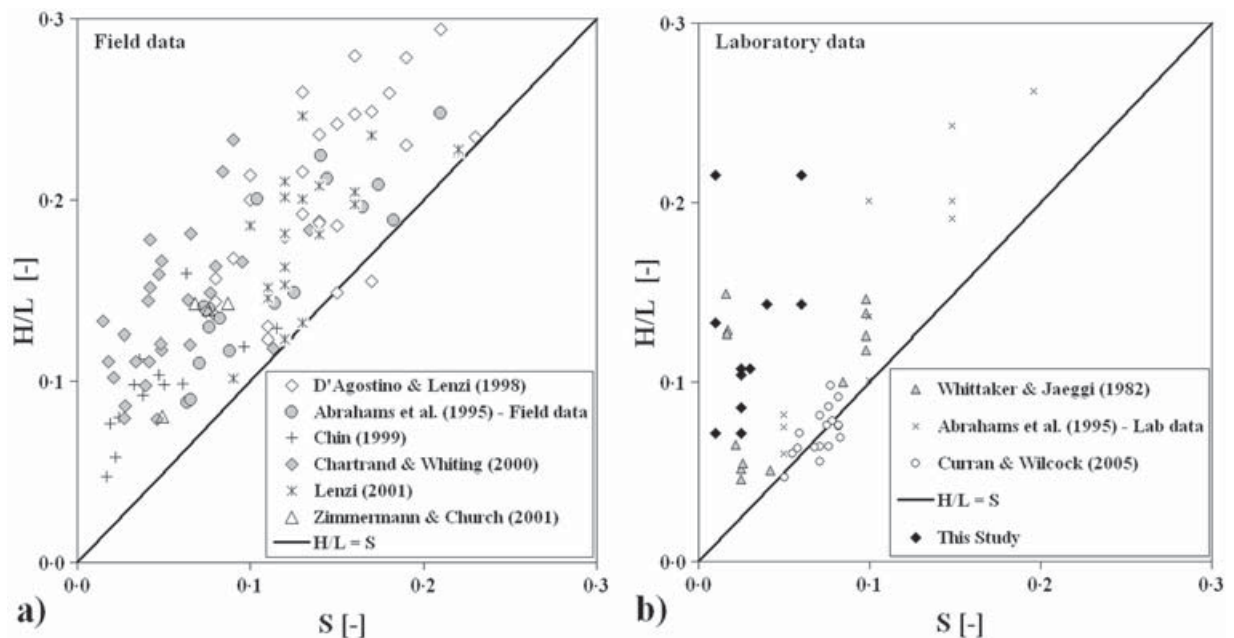
The hypothesis that step-pool streams are able to maximize flow resistance seems to be also confirmed by Figure 7(b), which shows that the  $(H/L)/S$  maximizing the laboratory flow resistance is the most common in natural step-pool sequences. This plot allows us to evaluate the effect of the slope on the flow resistance: it appears that almost all the experiments collapse on a single curve that has a maximum of the flow resistance when  $(H/L)/S$  ranges about two, in agreement with Abrahams *et al.* (1995), suggesting that when step-pool sequences are characterized by  $1 < (H/L)/S < 2$  the flow resistance is maximum. Moreover, note that the variance around the maximum shown by the field data in Figure 7(a) is much smaller in Figure 7(b), where the slope is introduced. Notwithstanding that field studies show considerable random variation of  $H/L$  (e.g. Curran and Wilcock, 2005), the latter finding suggests  $H/L$  is a non-linear function of  $S$ ; indeed it appears that  $(H/L)/S$  is not constant but is characterized by some distribution with a maximum between unity and two.



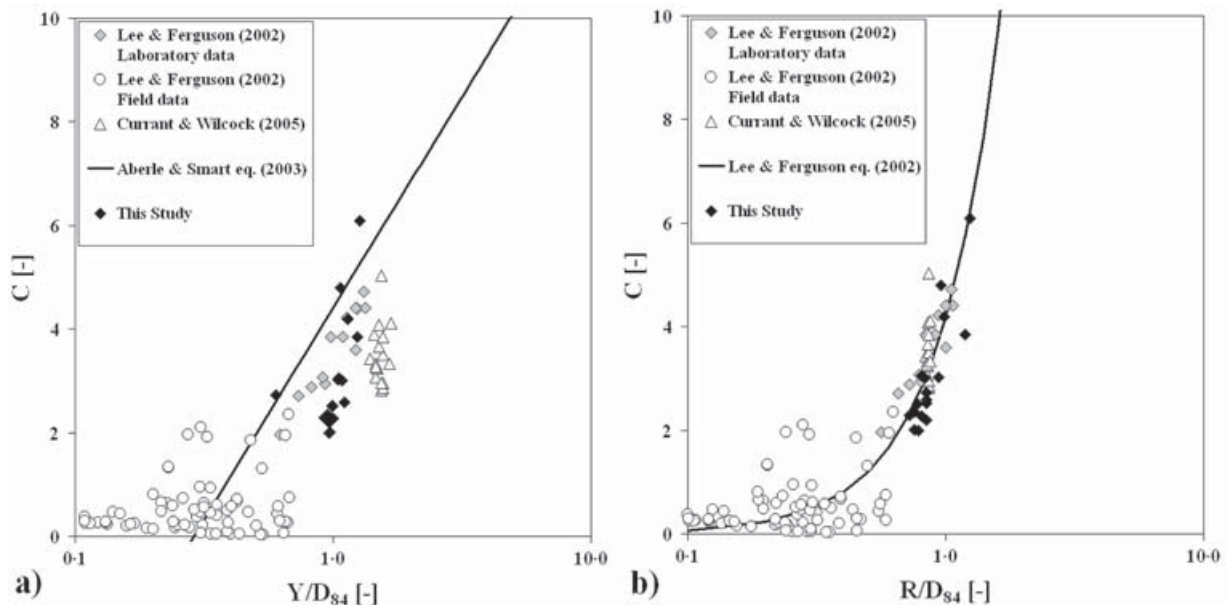


**Figure 7.** Statistical distribution of the ratio of step height  $H$  to step spacing  $L$  observed in natural step-pool beds (a) and comparison with laboratory results (b).

Geometric characteristics of employed natural step-pool sequences and flume pattern are respectively compared in Figure 8(a) and (b), showing the local step-pool gradient  $H/L$  as a function of channel slope  $S$ . Although the present experiments are characterized by flume slopes at the lower end of the range for natural step-pool sequences (Montgomery and Buffington, 1997), it appears that the present flume ( $S$ - $H/L$ ) pairs fall in the same range as real step-pool sequences. The figure also shows the line for  $H/L = S$ . The fact that all points are above this line indicates that all the data represent mature step-pool sequences with well scoured pools.



**Figure 8.** Field (a) and flume (b) measurements of step spacing  $L$ , step height  $H$  and channel slope  $S$ . The line represents  $H/L = S$ .



**Figure 9.** Flow resistance as a function of ratio of water depth  $Y$  to sediment size  $D_{84}$  in this study and flume and field data.

In Figure 9(a) the dimensionless Chezy coefficient  $C$  is shown as a function of relative submergence  $Y/D_{84}$  for the present experiments and for flume experiments with self-formed step–pool sequences of other authors (Lee and Ferguson, 2002; Curran and Wilcock, 2005).  $D_{84}$  is here referred to the steps only, and in the present experiments is assumed to be equal to the pebble axis  $D_b$ . In the case of the work of Curran and Wilcock  $D_{84}$  of the step is not known, and in the present analysis  $D_{84}$  of the bulk mixture is employed. Data are also compared with the logarithmic equation of Aberle and Smart (2003) formulated in terms of the ratio of water depth to the surface  $D_{84}$ , and obtained for laboratory step–pool sequences under various flow conditions and coarse sediment mixtures. Although Figure 9(a) shows a general common behaviour of the Chezy coefficient as a function of the ratio  $Y/D_{84}$ , the formula of Aberle and Smart does not seem to provide the best representation for these data. Following this finding, in Figure 9(b) the coefficient  $C$ , for the same data as employed in Figure 9(a), is shown as a function of the ratio  $R/D_{84}$ , where  $R$  is the hydraulic radius. In the case of the field data of Lee and Ferguson (2002),  $R$  is given by the authors; for the laboratory data  $R$  is calculated employing the approach proposed by Vanoni and Brooks (Vanoni, 1957) for the decomposition of boundary shear stress between wall and bed flumes. The plot shows that the data from the present experiments and those from other authors are in good agreement, and are well described by the formula of Lee and Ferguson (2002). Comparing Figure 9(a) and (b) it appears that the data presented here are much less scattered when described in terms of  $R/D_{84}$  rather than  $Y/D_{84}$ .

The formula of Lee and Ferguson (2002) reads

$$C = 4.186 \left( \frac{R}{D_{84}} \right)^{18}. \quad (12)$$

The plots in Figure 9 suggest that the present stripe pattern can reasonably be considered as a flume model of a step–pool sequence characterized by analogous geometry and dissipative behaviour.

We now investigate the possibility to study the flow resistance associated with natural step–pool patterns employing the results obtained for the present flume experiments. Unfortunately, notwithstanding the relatively wide availability of geometrical data about step–pool sequences in the literature, very few investigations have reported hydraulic quantities such as mean flow depth or mean flow velocity. This is probably due to difficulties of measuring low-frequency high-magnitude formative discharges, especially when step–pool sequences occur in rugged, mountainous and often inaccessible terrain (Chin, 1999). To overcome this lack, in the present work we have estimated the relative submergence occurring during a formative flood by applying Shields' (1936) criterion for the onset of sediment motion of the sediments representative of the step height (Grant, 1997). In this case the ratio  $R/D_{84}$  is given by

$$\frac{R}{D_{84}} = (s_g - 1) \frac{\tau_{cr}^*}{S} \quad (13)$$

where  $s_g$  is the specific gravity, assumed equal to 2.65, and  $\tau_{cr}^*$  the critical dimensionless shear stress for the onset of sediment motion, assumed to be around 0.045 in agreement with Chartrand and Whiting (2000) and Lenzi (2001).

Estimating flow resistance with the formula (12) of Lee and Ferguson (2002) and employing (13) it is found that

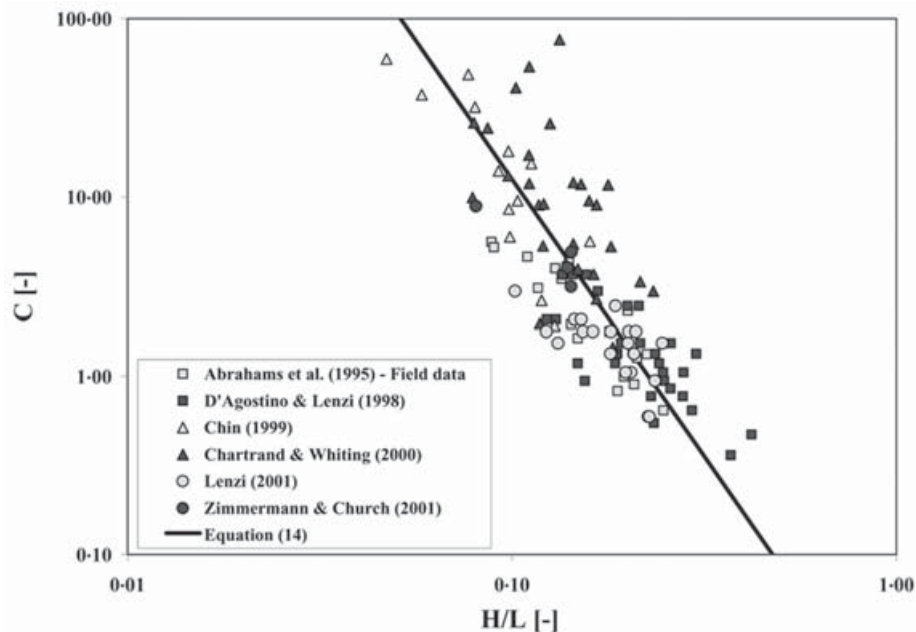
$$C = 4.186 \left( (s_g - 1) \frac{\tau_{cr}^*}{S} \right)^{18} \quad (14)$$

This equation allows one to estimate flow resistance in step-pool streams from geometric data only at conditions of incipient motion for  $D_{84}$ .

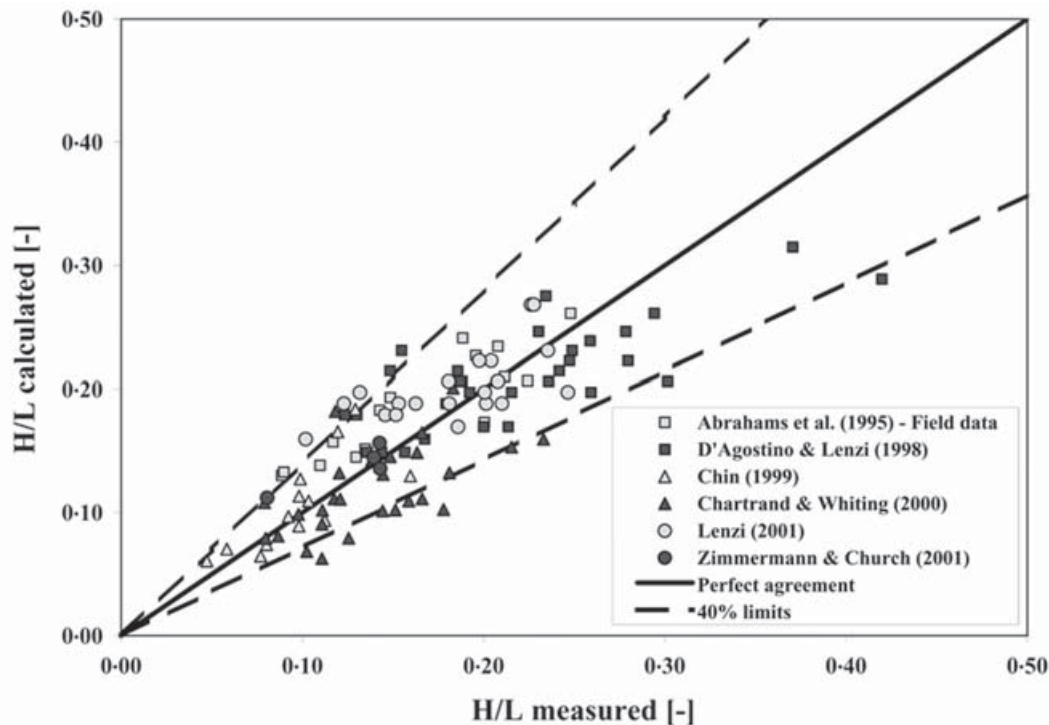
Equation (14) is then applied to step-pool sequences measured in the field by various authors (Abrahams *et al.*, 1995; D'Agostino and Lenzi, 1998; Chin, 1999; Chartrand and Whiting, 2000; Lenzi, 2001; Zimmermann and Church, 2001) to evaluate the dimensionless Chezy coefficient. Results are shown in Figure 10 as a function of  $H/L$ ; note that here the Chezy coefficient evaluated with Equation (14) is plotted as a function of  $H/L$  instead of  $(H/L)/S$  to avoid self-correlations in  $S$ . Laboratory data of other authors (Whittaker and Jaeggi, 1982; Curran and Wilcock, 2005) and the present flume data are also shown. It appears that both the field data and the present flume data give rise to a similar flow resistance for analogous values of  $H/L$ . Since the present flume data are those maximizing flow resistance, this result seems to suggest that step-pool channels evolve towards a morphology that develops maximum flow resistance as suggested by many authors (e.g. Davies and Sutherland, 1980; Whittaker and Jaeggi, 1982; Abrahams *et al.*, 1995). This feature may be explained as the tendency of the stream bed to reach a condition of maximum stability. This behaviour may be due to the fact that in this condition much of the energy that might otherwise be available for sediment erosion is lost (Ashida *et al.*, 1976). From Figure 10 a correlation between the dimensionless Chezy parameter  $C$  and the local step-pool gradient  $H/L$  appears; this correlation is expressed by the following equation:

$$C = 0.01 \left( \frac{L}{H} \right)^{3.1} \quad (15)$$

obtained by means of a regression on the field data ( $R^2 = 0.67$ ). Equation (15) considers geometric quantities only, thus suggesting that, at least in the case formative discharges, flow resistance in step-pool streams is mainly due to the



**Figure 10.** Flow resistance as a function of ratio step height  $H$  to step spacing  $L$  in various field studies.



**Figure 11.** Comparison between the ratio of step height  $H$  to step spacing  $L$  predicted by (14) and field values reported by various authors.

form-induced component of the resistance associated with the plunging jump over the step into the pool, whereas the surface-induced component of the resistance associated with relative roughness in the pool seems to play a much smaller role at least when a formative discharge occurs. Equation (15) gives an estimation of the flow resistance in the absence of hydraulic quantities such as the flow depth  $Y$ ; in this regard this equation can be employed to estimate the Chezy coefficient associated with high, nearly formative, discharges, by means of geometrical quantities only, which are much easier to measure.

Combining Equations (14) and (15), the following relationship between steepness and slope is found:

$$\frac{H}{L} = 0.646S^{0.581} \quad (16)$$

suggesting that when  $S$  is between 7 and 30%,  $H/L$  is between 0.139 and 0.321 and therefore  $(H/L)/S$  is ranging between 1.07 and 1.97; these values are in agreement with the observations by Abrahams *et al.* (1995). Note that Equation (16) predicts a power relationship between  $S$  and  $H/L$ . This finding is in agreement with those of Zimmermann and Church (2001) and Comiti (2003), suggesting that  $(H/L)/S$  is a function of  $S$  at least in the case of relatively low slopes.

Finally, a comparison between  $H/L$  predicted by Equation (16) and field values reported by various authors is presented in Figure 11, showing that almost all points are contained in the  $\pm 40\%$  limits. Notwithstanding the different measuring techniques between different investigations, this result appears fairly encouraging and seems to confirm the validity of the present approach.

## Conclusions

A theoretical analysis of experimental data obtained by means of an extensive laboratory activity using a schematic arrangement of macro-roughness represented by a regularly spaced stripe pattern has shown that when the macro-roughness is positioned according to an 'optimal' spacing value, ranging around 10 times the stripe height, flow resistance is maximum.

A statistical analysis of a large number of geometrical data about step-pool sequences observed in the field and reproduced in the laboratory suggests that this 'optimal' spacing is close to what is frequently encountered in natural streams. A comparison between the dissipative behavior of real step-pool sequences reproduced in the laboratory and the present flume experiments suggests that the pattern of macro-roughness disposition associated with the 'optimal' spacing value is able to reproduce the fundamental mechanisms occurring in the development of flow resistance in step-pool sequences. Such findings suggest that step-pool streams evolve towards a morphology that develops maximum flow resistance, therefore achieving maximum stability of the bed (Abrahams *et al.*, 1995).

In order to overcome the difficulties of measuring hydraulic variables in step-pool streams, we provide a criterion to estimate flow resistance in the case of high formative discharges based on geometric quantities only. This criterion predicts that the step-pool pattern develops maximum flow resistance when  $1 < (H/L)/S < 2$ , in agreement with the observations of Abrahams *et al.* (1995).

Further developments are devoted to understanding the reason for the presence of the maximum of flow resistance in step-pool sequences: a stability analysis of the bed to perturbations of the type associated with a step-pool pattern might provide some useful insight into this phenomenon.

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